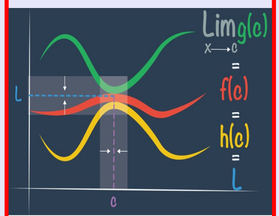
Calculus I

Lecture 52



Feb 19-8:47 AM

Given $f(x) = \int_{1-2x}^{1+2x} t \sin t \, dt$

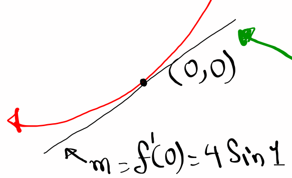
1) Find $f(0) = \int_{1-2(0)}^{1+2(0)} t \sin t \, dt = \int_{1}^{1} t \sin t dt = \boxed{0}$

2) Find $f'(x) = (1+2x)\sin(1+2x)\cdot 2 - (1-2x)\sin(1-2x)\cdot -2$

$f'(x) = 2(1+2x)\sin(1+2x) + 2(1-2x)\sin(1-2x)$

3) Find $f'(0) = 2(1)\sin 1 + 2(1)\sin 1 = 4\sin 1$

$f(x) = \int_{1-2x}^{1+2x} t \sin t \, dt$



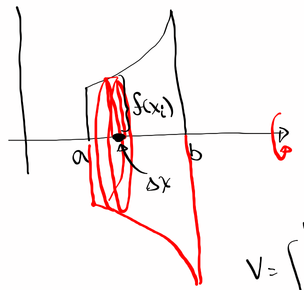
Eqn of tan. line

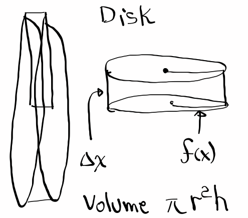
$y - 0 = 4\sin 1 (x - 0)$

$\boxed{y = 4\sin 1 \cdot x}$

May 20-8:45 AM

Suppose $f(x) \geq 0$ and cont. on $[a,b]$, we take the region enclosed by $f(x)$ & x-axis on $[a,b]$ and revolve it by x-axis.





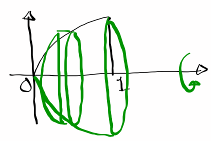
Volume $\pi r^2 h$

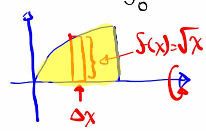
$r = f(x)$ $\quad h = \Delta x$

$$V = \int_a^b \pi [f(x)]^2 \, dx$$

Rotate the region bounded by $f(x) = \sqrt{x}$, $y = 0$, $x = 0$, and $x = 1$.

$$V = \int_0^1 \pi [\sqrt{x}]^2 dx = \pi \int_0^1 x \, dx = \pi \cdot \frac{x^2}{2} \Big|_0^1 = \boxed{\frac{\pi}{2}}$$

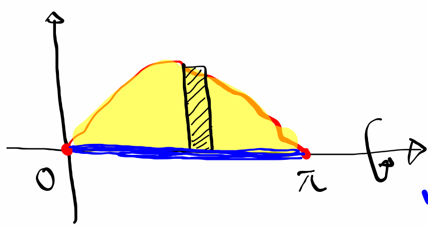




May 20-8:53 AM

Consider the region enclosed by x-axis, $f(x) = \sqrt{\sin x}$ from $x = 0$ to $x = \pi$.

Rotate by x-axis, Find the volume.



✓ 1) Ref. Rect. ⊥ to A.O.R.
2) Region is totally attached to A.O.R.

Disk Method

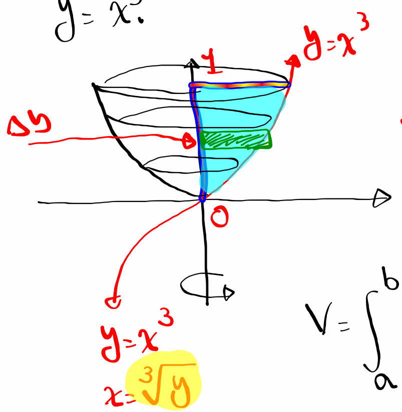
$$V = \int_a^b \pi [f(x)]^2 dx = \pi \int_0^\pi \left(\sqrt{\sin x}\right)^2 dx = \pi \int_0^\pi \sin x \, dx$$

$$= \pi \cdot -\cos x \Big|_0^\pi = -\pi[\overset{-1}{\cancel{\cos \pi}} - \overset{1}{\cancel{\cos 0}}] = \boxed{2\pi}$$

May 20-9:02 AM

Consider the region enclosed by $x=0$, $y=1$, and $y=x^3$?

Let's rotate by Y-axis



1) Ref. Rec. $\perp$ A.O.R.
2) Region is totally attached to A.O.R.

Disk Method

$$V=\int_a^b \pi\,[f(y)]^2\,dy$$

$$=\int_0^1 \pi\left[\sqrt[3]{y}\right]^2 dy = \pi\int_0^1 y^{2/3}\,dy$$

$$=\pi\cdot\frac{y^{5/3}}{5/3}\Big|_0^1 = \frac{3\pi}{5}\,y^{5/3}\Big|_0^1 = \frac{3\pi}{5}$$

May 20-9:09 AM

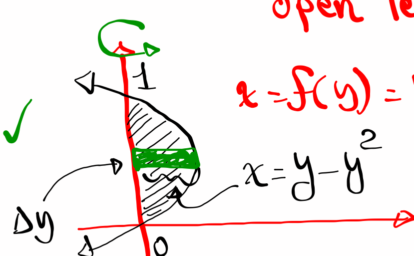
Rotate the region bounded by $x=y-y^2$ and $x=0$ by y-axis. Find its volume.

sideway Parabola open left

$x=f(y)=y-y^2$

1) Ref. Rect. $\perp$ A.O.R. ✓
2) Region Totally attached to A.O.R. ✓

Disk Method



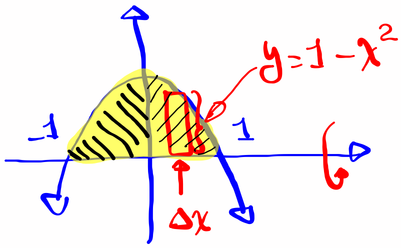
$$V=\int_0^1 \pi\,[f(y)]^2\,dy = \pi\int_0^1 (y-y^2)^2\,dy = \pi\int_0^1 \left[y^2-2y^3+y^4\right]dy$$

$$=\pi\left[\frac{y^3}{3}-\frac{2y^4}{4}+\frac{y^5}{5}\right]\Big|_0^1 = \pi\left[\frac{1}{3}-\frac{1}{2}+\frac{1}{5}\right] = \frac{1\pi}{30}$$

$$=\boxed{\frac{\pi}{30}}$$

May 20-9:17 AM

Find the volume when the region enclosed by $y = 1 - x^2$ and $y = 0$ rotated by x-axis.



✓ 1) Ref. Rect. ⊥ A.O.R.

✓ 2) Region is completely attached To A.O.R.

Disk Method

$$V = \int_a^b \pi [f(x)]^2 dx = \int_{-1}^{1} \pi \underbrace{(1-x^2)^2}_{\text{even Function}} dx = \pi \cdot 2\int_0^1 (1 - 2x^2 + x^4)\, dx$$

$$= 2\pi \left[x - \frac{2x^3}{3} + \frac{x^5}{5} \right]\Big|_0^1 = 2\pi\left(\frac{8}{15}\right) = \boxed{\frac{16\pi}{15}}$$

May 20-9:26 AM

Consider the region bounded by $y = \sqrt{25 - x^2}$, $y = 0$, $x = 2$, and $x = 4$.

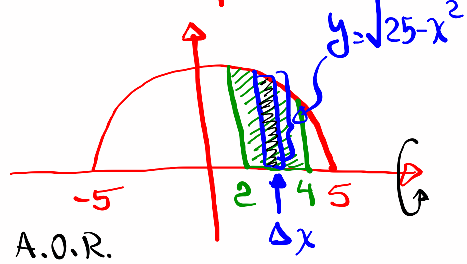
$x^2 + y^2 = 25$

Circle

Top half

Rotate by x-axis.

Find the volume.



Disk Method

1) Ref. Rect. ⊥ A.O.R.

2) Region is totally attached to A.O.R.

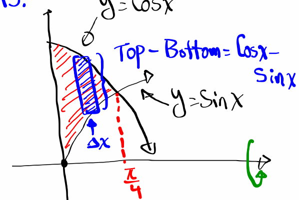
$$V = \int_2^4 \pi \left[\sqrt{25-x^2}\right]^2 dx = \pi \int_2^4 (25 - x^2)\, dx = \pi \left[25x - \frac{x^3}{3}\right]\Big|_2^4$$

$$= \boxed{}$$

May 20-9:36 AM

Rotate the region bounded by $y=\sin x$, $y=\cos x$ on $[0,\frac{\pi}{4}]$ about x-axis.

Find the Volume.



Ref. Rect. ⊥ A.O.R.

Region is not totally attached to A.O.R.

we cannot use disk → we should use washer Method.

Washer Method

$$V=\int_a^b \pi\left[\text{Top}^2-\text{Bottom}^2\right]dx$$

$$=\int_0^{\pi/4}\pi\left[\underbrace{\cos^2 x-\sin^2 x}_{\cos 2x}\right]dx=\pi\int_0^{\pi/4}\cos 2x\,dx$$

$u=2x$

$du=2dx$

$\frac{du}{2}=dx$

$x=0 \to u=0$

$x=\pi/4 \to u=\frac{\pi}{2}$

$$=\pi\int_0^{\pi/2}\cos u\cdot\frac{du}{2}$$

$$=\frac{\pi}{2}\cdot\sin u\Big|_0^{\pi/2}=\frac{\pi}{2}(1-0)=\boxed{\frac{\pi}{2}}$$

May 20-9:43 AM